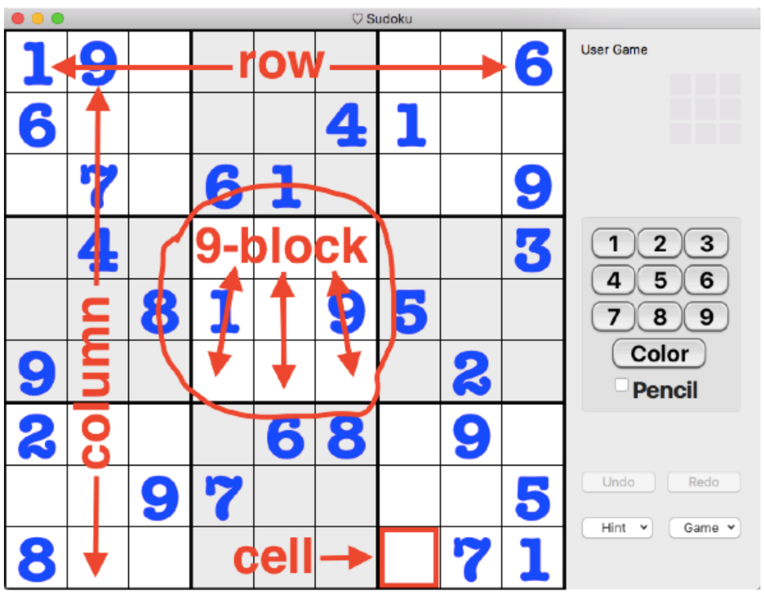
# Dale's Sudoku De-frustrater



You've tried it before but each time you give up in frustration. You are fine with the easier puzzles. But you grind to a halt when the number of empty cells starts to exceed the number of pre-filled cells. Like anything else there's a trick to doing sudoku.

Let's start at the beginning and assume nothing. The basic concept of sudoku play is easily learned.

First, a bit of terminology:




Just as in a spreadsheet, we'll refer to the small boxes as cells. A cell is either empty, like the one marked in red, or contains a single digit.

The nine rows and nine columns each containing nine cells are obvious. Row 1 and column 2 are marked in red.

The thicker black lines sub-divide the grid into nine squares of nine cells each. Terminology for these larger squares varies, but we'll call them 9-blocks, since they're blocks of nine cells. I've circled the centre 9-block in red, above. To its left is the middle-left 9-block (middle row, left column). Beneath it is the middle-bottom 9-block, etc.

Next, there are three simple **rules**:

Rule 1: Each row must contain one and only one instance of each integer from 1 to 9.
Rule 2: Each column must contain one and only one instance of each integer from 1 to 9.
Rule 3: Each 9-block must contain one and only one instance of each integer from 1 to 9.

(Many sudoku puzzle creators also obey an elegant Rule 4: the pre-filled cells must be arranged symmetrically.)

There is but one **goal**: Fill in all empty cells according to the three rules.

Finally, the basic **technique** for solving (filling in all empty cells) is equally simple: Systematically probe for weaknesses by using all three rules.

Now consider the following puzzle:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 9 | | | | | | | 6 |
| 6 | | | | | 4 | 1 | | |
| | 7 | | 6 | 1 | | | | 9 |
| | 4 | | | | | | | 3 |
| | | 8 | 1 | | 9 | 5 | | |
| 9 | | | | | | | 2 | |
| 2 | | | | 6 | 8 | | 9 | |
| | | 9 | 7 | | | | | 5 |
| 8 | | | | | | | 7 | 1 |

It’s ranked as “hard” in difficulty. Only 28 of its 81 cells are pre-filled, so a snap it’s probably not going to be. In a daily newspaper it might show up on a Friday or even a weekend.

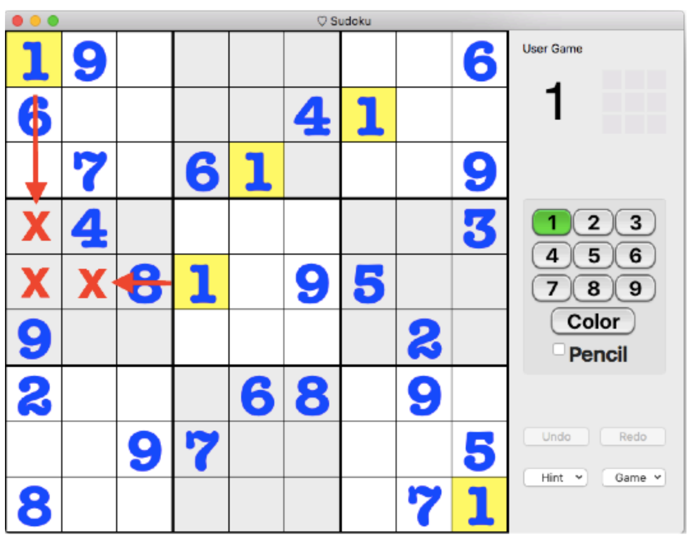
To follow along, either print this page, draw the puzzle on a sheet of paper or enter it into a Sudoku app on a computing device. (I use the excellent Mac and IOS app, Heart Sudoku, that you see in the illustrations herein. Simple Sudoku on Windows PCs in equal excellent.) This tutorial will give you the techniques you need to solve this puzzle. But if you want the information to stick, work your way to the solution by completing each step on your own.

## 1. Standard operating procedure: decimation by elimination

We start by considering just the 1s with a focus on 9-blocks and rule 3:

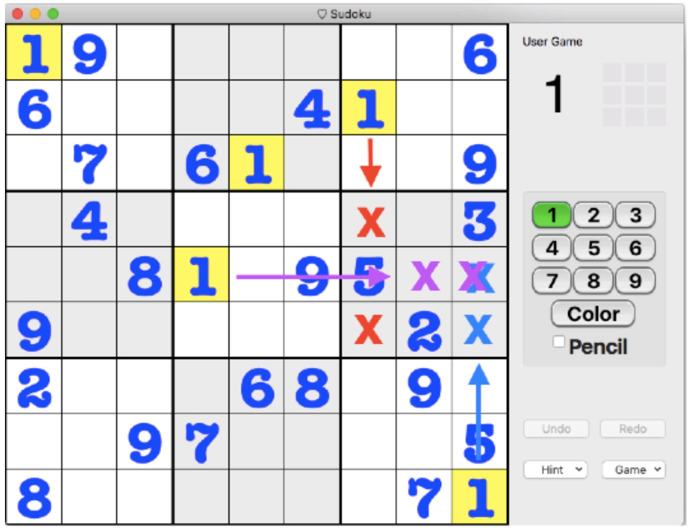

(Software sudoku apps are handy. They typically highlight all occurrences of a given digit.)

The top-left, top-middle and top-right 9-blocks each already contain a 1, so we skip them as per rule 3. Moving to the middle row, the middle-left 9-block does not yet contain a 1.



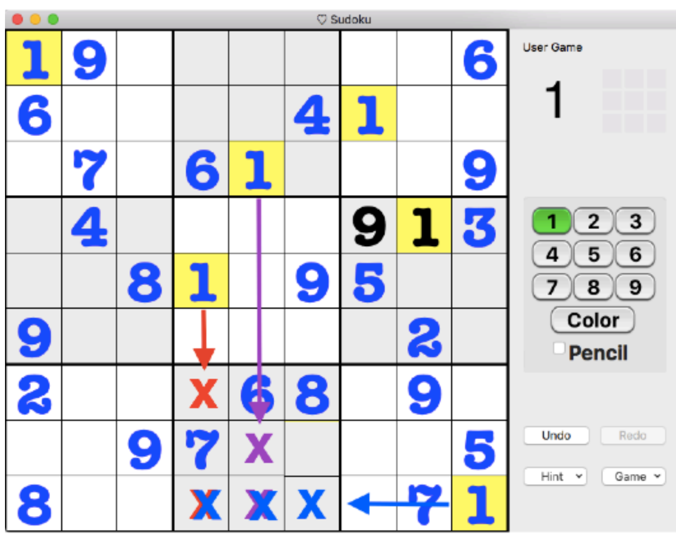

There are two pre-filled 1s that bear upon the middle-left 9-block. As per rules 1 and 2 either one would serve to eliminate the digit 1 as a candidate for the three cells shown with red X's. But that leaves three of the six empty cells in that 9-block still empty, as shown. What we need is the opposite: five of six cells eliminated. So no joy here.

The centre 9-block already contains a 1.



Moving on, the middle-right 9-block is the focus of three 1s at the same time. Above, I've used coloured arrows and x's to illustrate how we can use rules 1 and 2 to eliminate all but one cell in the middle-right 9-block from consideration. Therefore, we can click on that one remaining cell then enter the digit 1 there.

We use the same logic to enter a 1 in one of the three remaining 9-blocks on the bottom row:



Eliminating all but one cell is the bread-and-butter tool for sudoku solving. In this case we used the rule 3 requirement that there can only be one digit 1 in any given 9-block. Systematically work through the other digits — 2, 3, 4, etc — looking for 9-blocks with digits to fill in. I found two more digits to fill in that way. Hopefully, you can too.

But the same logic also applies when we can eliminate all but one empty cell in a row or in a column (rules 1 and 2). For example, when we get around to checking the 9s row by row, we get:


We can place a 9 in row four.

Next, we start over again going through the digits this time looking at 9-blocks, rows and columns. Immediately, this results in action, as we look at the 1s. In fact, we can place the final two missing 1s at this time. Here’s the puzzle at this point in the solution:


And here’s the decimation by elimination tactic stated succinctly:

> **When we can eliminate all but one of the empty cells in a row, column or 9-block that can contain a given digit, the remaining empty cell** *does* **contain that digit.**

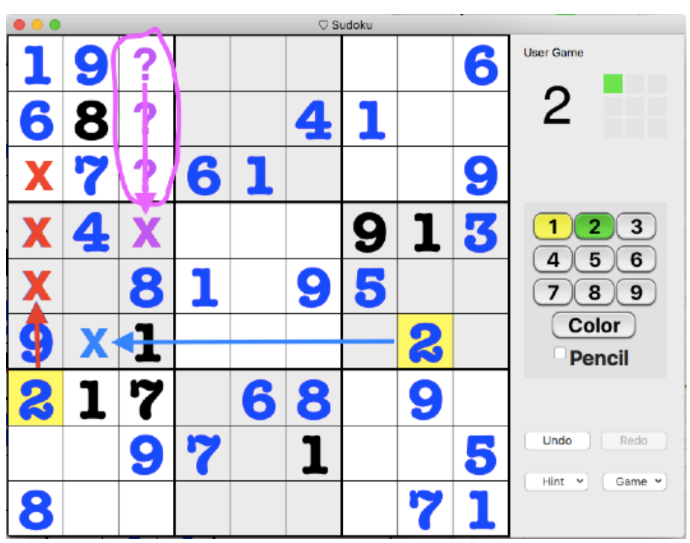
We could go on to solve this entire puzzle using just this one tactic. Doing so would be an excellent way to practice. But It’s time to learn a new decimator. So we’ll stick with the current state of the puzzle and move on.

## 2. Decimation by devious deduction

The digit 2 in the leftmost column eliminates one empty cell in the top-left 9-block as a possible digit 2 location. It also leaves three empty cells to form a vertical row in that same 9-block. We don’t know which one of them contains a 2. We do know that one of them must:


Crucially, rule 2 tells us that *no other empty cell in that column* can contain the digit 2. That’s our devious deduction. The purple **2?**s in the above illustration indicate this point. One of them has to contain a digit 2 or the upper-left 9-block will not have any 2 at all. But equally the implication of that is that a digit 2 cannot go any where else in column 3. It’s an invaluable bit of logic that pays endless dividends in the world of sudoku

Now we look at the middle-left 9-block. The devious deduction above eliminates one of the empty cells in this 9-block (purple X). The two pre-filled 2s eliminate three of the four remaining empty cells in it (red and cyan X’s). So we can fill in its middle cell with a 2:




See if you can locate another devious deduction to place another digit before turning the page.

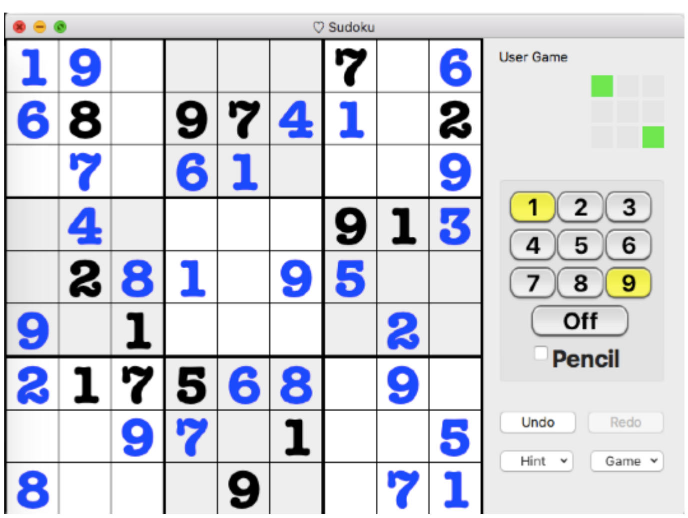
Here is another devious deduction example, this time using 5’s:

This slightly devious bit of reasoning we’re using is absolutely crucial to solving any but the simplest tier of sudoku games. Here, the two question marks in the bottom-left 9-block eliminate all three bottom cells in the bottom-centre 9-block, leaving only once possible cell to contain a 5. And of course we can use it in columns as well as in rows (as we’ll see below).

Here is the concept. Engrave it in your memory. It will serve you well:

**When there are only two or three empty cells in a 9-block that can contain a given digit and they lie in a straight line, they serve to eliminate any other occurrence of that digit in the rest of the row or column they occupy.**

Using just what we already know, do some more work on the puzzle until you get to this state:



And now it’s once again time to try something different.

## 3. Decimation by double-cross elimination

Grinding through the digits in numerical order is bread-and-butter. But occasionally we also need to step back to look at the big picture. Look for any rows, columns and 9-blocks that have only one, two, or three empty cells in the puzzle picture above. (Of course, if there is only one empty cell, filling it in is trivial.)

The seventh row jumps out, having only two empty cells. Decimation by elimination allows us to quickly fill in the missing 3 and 4, completing row seven. The rightmost column now has only two empty cells. We can quickly fill them in by using elimination.

Doing so turns row five into one having only three empty cells:


Each of these 3 cells can only contain the digits 3, 4 and 5. But which ones go where?

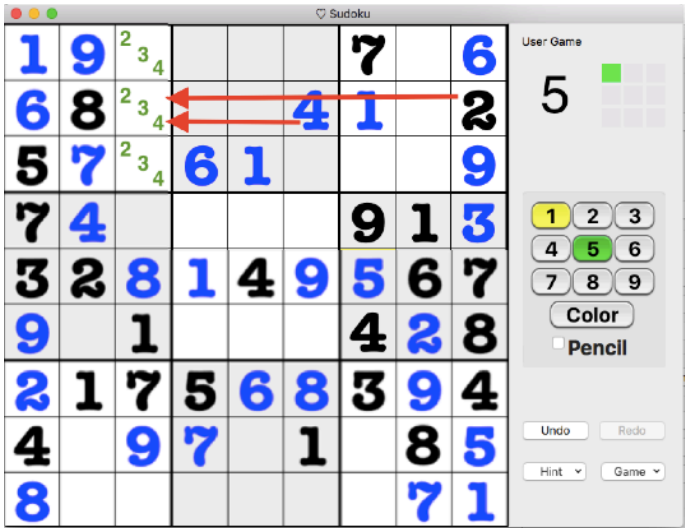
This is where the new bit of deviousness comes into play. In the rightmost cell of row five we can eliminate 3 as a possibility but not 4 or 6. In the centre empty cell we can eliminate 6 but not 3 or 4. But in the leftmost empty cell we can eliminate both 4 and 6, leaving 3 as the necessary solution for that cell:

From there we can fill in the centre cell with a 4, since it can no longer contain a 3 and a 6 is pointing at it. That leaves the rightmost cell as being the 6 and the centre row is complete.

Getting back to the puzzle, now the leftmost column has only three empty cells. Simple elimination takes care of them. The middle-right 9-block has that easy-pickings single empty cell that's ripe for the picking. And the puzzle now looks like this:

The middle-left 9-block has only two empty cells, begging to be completed. We know one must be 5 and the other 6. But frustratingly, nothing tells us which is which, so we ignore it and move on. Same thing for the bottom-right 9-block. But we're now in a situation to employ the double-cross again:

The three-in-a-row empty cells in the top-left 9-block that were so useful in the previous section are still awaiting assignment of the digits 2, 3 and 4. Since two of those three digits can be found, both pointing at one of these cells…



…we deduce that it must contain the third digit, a 3.

Turning this into a tactic we have:

**When there are three empty cells in a row, column or 9-block, and when we can eliminate two of the three candidate digits for one of those empty cells, then it must contain the third digit.**

Situations that require this ploy don’t arise quite as often as is the case for the previous two principles. But when they do arise they’re invaluable.

And that’s it. We’ve gone from the 28 pre-filled cells to 51. There remains much to do, but the rest of the puzzle can be solved using just the above tactics. You should now be able to complete this puzzle on your own.

I think you’ll find that the more of a sudoku you fill in the more you can accomplish by a big picture scan for two and three empty cells in a row, column or 9-block. And therefore the less often you’ll need to grind through digit by digit.

From time to time, despite these powerful tools, you’ll get stuck. It will look as though there’s no way to move forward. At that point turn your attention away from the puzzle to something else entirely. Then after as little as a few minutes come back to the puzzle. Chances are good that your little grey cells have been working away and will have an *a ha!* waiting to show the way.

## Moving forward

Just these three basic ploys, together with a reasonably methodical approach, will get you through an awful lot of sudoku. Make sure the logic behind each ploy is crystal clear in your mind so you will recognize every situation in which it occurs. Practice solving dozens of sudoku puzzles of similar difficulty so recognizing when each tactic applies becomes easy-peasy. In case you don’t have one handy, just below is a second puzzle to practice on. Or you could look at it as your final exam for this course. ;) It can be solved using only the techniques you’ve already learned:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | | 8 | | | | 2 | | |
| 3 | | 5 | | 9 | | | 6 | |
| | | | | 7 | | | | 9 |
| | | | | 1 | | | 9 | 5 |
| | | 6 | 2 | | 7 | 3 | | |
| 1 | 3 | | | 8 | | | | |
| 7 | | | | 3 | | | | |
| | 5 | | | 2 | | 9 | | 8 |
| | | 9 | | | | 6 | | 4 |

Still, there remain dozens yet more devious techniques to employ when even more diabolical puzzles call for them. Do an Internet search for sudoku techniques and learn about hidden doubles, hidden triples, x-wings, skyscrapers plus many, many more testaments to human ingenuity exercised in a good cause. I cover some of the most useful of these in a companion tutorial. But I urge you to leave that until you’ve spent weeks or months practicing this tutorial’s techniques.

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Final note: you may have trouble finding references to decimation by elimination. That’s because it’s usually more prosaically referred to as finding a *hidden single*. Similarly, devious deductions are usually less fancifully known as *locked candidates*. (*Candidate* is sudoku tutorial speak for a possible answer for an empty cell.) I don’t know a prior name for our double-cross elimination but there’s probably one out there.

And here’s a bonus puzzle:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 1 | | | | | | | |
| 8 | | | 7 | | 2 | | 9 | 3 |
| | | | 5 | | | | 4 | |
| | 4 | 5 | | 2 | | | | |
| | 7 | | 6 | | 5 | | 3 | |
| | | | | 4 | | 2 | 7 | |
| | 2 | | | | 4 | | | |
| 5 | 3 | | 1 | | 6 | | | 7 |
| | | | | | | | 5 | 1 |

You can completely solve this puzzle for three of the nine digits in the first pass and solve completely in three passes.