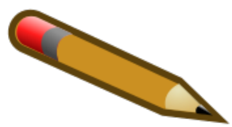
Dale’s Sudoku De-frustrater 2

Note: This is the sequel to my first sudoku tutorial. The three tactics in that tutorial are the bread and butter of daily sudoku play. I strongly recommend putting in a lot of sudoku practice with that material before proceeding.

1. Pencil marks permitted



When solving a difficult sudoku it is often useful to write small numbers in a cell when you’ve reduced the possibilities for that cell down to a small number. These are called the *candidate* solutions to a given cell. When doing a sudoku on paper, such as a newspaper or sudoku book, we can literally use a pencil and eraser to make note of candidates. Many on-line sudoku’s and sudoku apps have an alternate input method, typically called pencil marks, just for the purpose of entering candidates.

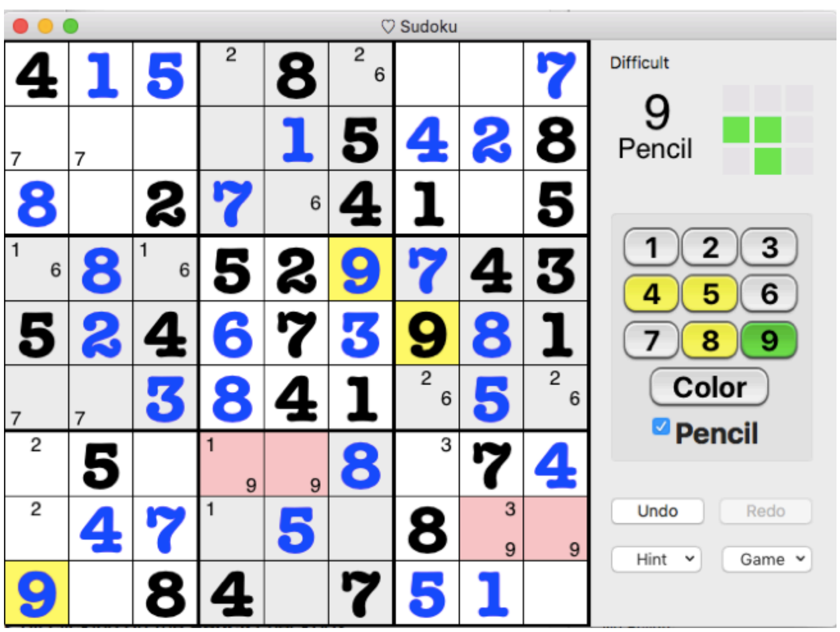


Fig. 1: Pencil marks in use

Here I’ve made a first pass through a puzzle filling in empty cells which have a clear single answer. (The big blue numbers are the givens; the big black numbers are my solutions.) Then I did a second pass, this time filling in more solved cells, but also filling in candidate pencil marks by first clicking on the **Pencil** checkbox to select that input mode.

One of the many advantages of using a sudoku app is that the pencil marks are arranged into the consistent pattern of 1, 3, 7 and 9 in the corners of a cell, 2, 4, 6, 8 in the edge middles and 5 in the centre. This is a good habit to get into if you're doing actual pencil marks on a paper puzzle. The *location* of the pencil mark, such as 6 in the middle-right edge of a cell, becomes as much a visual cue to the candidate as the actual digit, which may be too small to easily read.

Over the years I've worked out a set of rules I follow for entering pencil marks during early play:

1. I only enter pencil marks that are candidates in reference to the 9-block that contains them
2. I only enter pencil marks either when there are only two candidates for a given digit in a given 9-block, or
3. When there are three candidates in the 9-block for that digit that lie in a straight horizontal or vertical line.

So examining the upper-left 9-block in Fig. 1, the digits 3, 6, 7 and 9 are all awaiting placement. But there are three possible locations for 3s and these locations are not in a straight line. There are four possible locations for sixes, so no go. And again there are three possible locations for 9s that also do not form a straight line. But there are only two possible locations for 7s so I marked them in (and as a bonus they happen to form a straight line). Go through the same logic for each of the other 9-blocks. See if you don't agree with my result. Except, of course, for the 9s.

At this point I can enter two 9s as pencil marks in the upper-right 9-block. Doing so reveals a devious deduction elimination, since those two 9s are in the same column as one of the 9s in the bottom-right 9-block. So, at the very least, using pencil marks aids in spotting the three decimations we covered in the first tutorial.

But another point we can see from the screen shot above is how to understand rule 1. If we look at the sixth column we might be tempted to enter pencil marks for a 2 and a 6 in the top and second-from-bottom cells of that column. Problem is that in five minutes you'll be scanning the eight row or the bottom-middle 9-block, see that 2 or that 6 and make an incorrect move on the assumption that they are validly placed candidates in a row or 9-block context. In other words, pencil marks quickly lose meaning unless they are placed according to a rigidly consistent set of constraints. It would be very nice to also be able to make note of that 2-6 pairing in the eight column for future reference.

Note: towards the end of a game, when all else fails, I switch from adhering to the three rules approach and instead follow a second rule-based approach. I now fill in *all* possible candidates that can't be logically eliminated throughout the puzzle. This maintains logical coherence, so none of the pencil marks will be misleading. But it risks obscuring the presence of crucial patterns of digits. If you're working in a sudoku app that provides highlighting, that problem goes away.

2. Double-doubles banish toil and trouble

Here's another puzzle. I've made a first pass filling in obvious cells, followed by a second pass filling in pencil marks according to be above rules. I took this screen grab after filling in pencil marks for 1s through 8s:

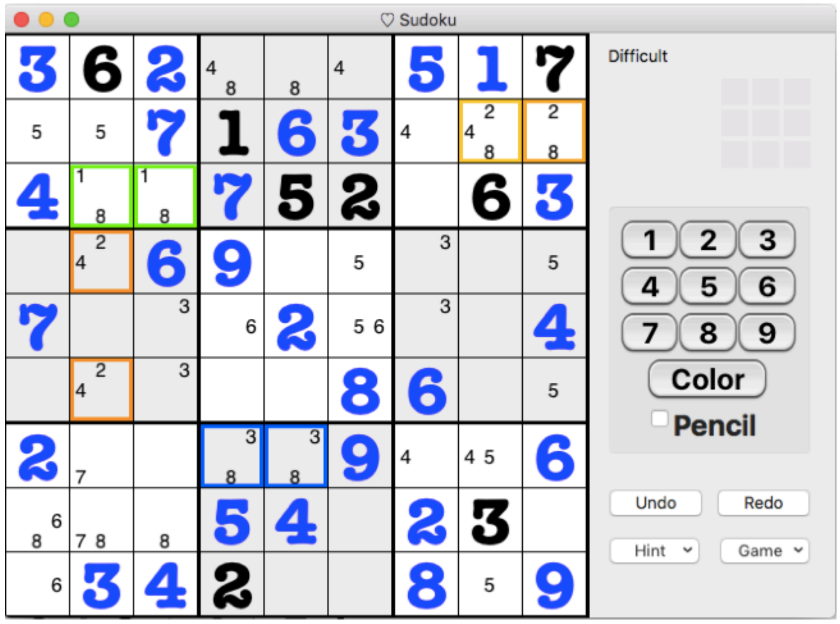


Fig. 2: Cells containing multiple double-doubles indicated (puzzle work-along 1)

One of the things a veteran sudoku enthusiast is always on the look-out for are two cells in a row, column or 9-block that have the same two candidates that can only appear in those same two cells. I'll call this configuration a double-double. In the above screen shot we see this illustrated by the 1-8 pairs in the upper-left 9-block, by the 2-8 pairs in the upper-right 9-block, the 2-4 pairs in the middle-left 9-block, and the 3-8 pairs in the bottom-middle 9-block. The upper-right 9-block shows why double-doubles are useful. The two cells with 2s and 8s penciled in can only contain either a 2 or an 8. We don't yet know which digit goes in which of these two cells. But the key is that we know *no other* digit can go in either cell. If ultimately the 2 belongs in the left of this pair of cells, then the 8 must go in the right cell. Likewise, If the 8 belongs in the left of this pair of cells, then the 2 must go in the right cell. Either way, no other candidate digit can go in those two cells. So the 4 candidate in the left cell of the 2-8 pair can be eliminated, leaving the other 4 in the same 9-block as the solution for the cell it's in.

Similarly, the 1-8 double-double in the upper-left 9-block tells us that 9s can only go in the two cells with 5s in that 9-block. And further, looking at the row the 1-8 double-double is in, the seventh cell in that row can only contain a 9. (But of course that was already clear after placing the 4 in the upper-right 9-block.)

In general, the double-double pair acts as *virtual* located digits for both its digits — but with a limited scope. Look at the 3-8 pair marked in blue in Fig. 2. We don't know which of those two cells will be the final location for those two digits. But as far as the rest of row containing the 3-8 pair is concerned it doesn't matter. Just as though we actually knew the 3's location, we can say that the digit 3 cannot go in any other cell in that row. The same is true for the 3-8 pair's 9-block. What the 3-8 pair can't do is help us with either of the two columns they live in.

Now that you're aware of the power of the double-double you'll quickly learn to keep an eye on any row, column or 9-block that contains one, given the leverage they provide. Prosaically, double-doubles are normally called *hidden doubles* (even though revealing them is one of the key strengths of pencil marks). Just as powerful but rarer and harder to spot are *hidden triples*.

If you get stuck in solving a particular sudoku, it's always wise to methodically scan each row, column, and 9-block for three cells that contain the only instances of three digits in that unit.

For example, in the following puzzle the 1s, 2s and 5s in the middle 9-block can only reside in the centre row:

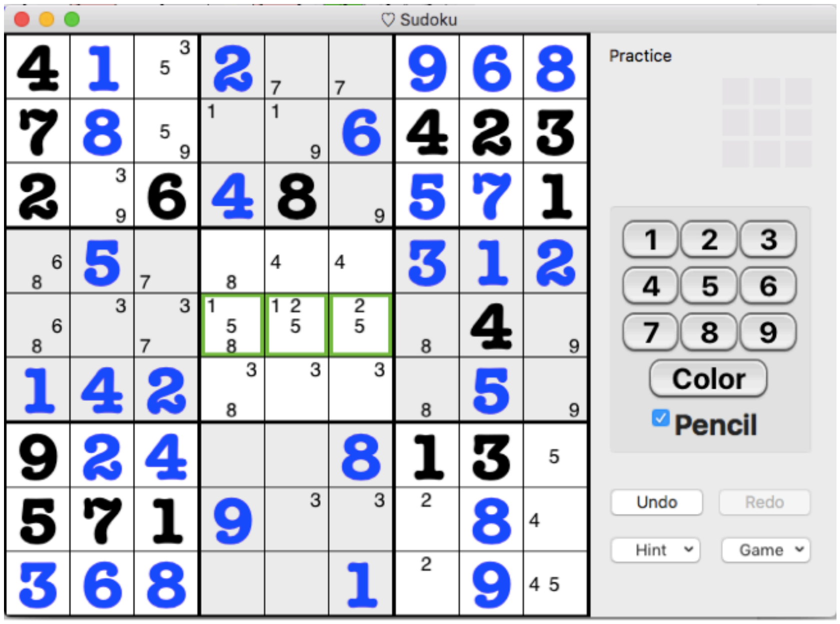


Fig. 3: Hidden 1-2-5 triple

So we can eliminate the 8 in both that row and 9-block.

Similarly, hidden quadruples are always a possibility — but thankfully are as rare as they are tedious to tease out. Here's one:

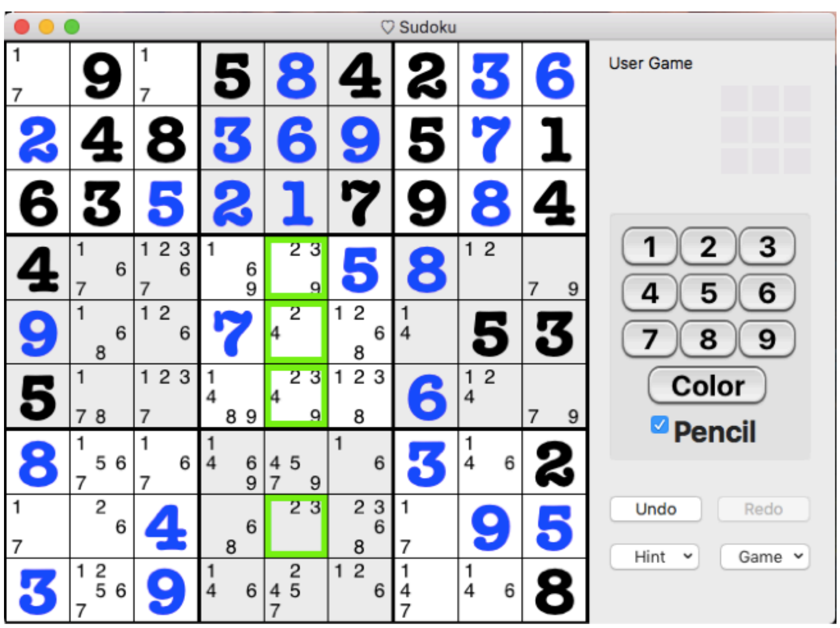


Fig. 4: Hidden 2-3-4-9 quadruple

About 20 minutes in, the multiple 2s, 3s and 4s caught my eye in the centre column. Then I saw the 9s could be included to form a quad. It was only after using that to eliminate all other 2s, 3s, 4s and 9s in the same column that I saw the hidden 5-7 double that was lurking there all along.

Summing these concepts into a rule, we get:

> **When any two empty cells in a row, column or 9-block can only contain the same two digits, they serve to eliminate any other occurrence of those digits in the rest of the row, column or 9-block they jointly occupy. Ditto for three cells containing the same three digits, etc.**

2. The X-wing sting

(courtesy PNGGuru.com)

There are end-of-week difficulty puzzles you’ll encounter that require yet another technique to crack. Continuing work on the puzzle back in Fig. 2, I got to this point:

Fig. 5: X-wing on 1s indicated (work-along puzzle 1)

> **Note**: follow along with this and other example puzzles by printing, copying or inputting the starting layouts provided in the appendix at the end of this document.

In Fig. 5 I’m ready for another pass through the 1s. Looking at the middle-left 9-block, we’d normally have to conclude there’s nothing to do. It looks as though 1s can go in all five of the unsolved cells in that block. Here’s where the new trick comes in. There are two 1s in both the second and third columns, and these columns intersect the middle-left 9-block. Here we have the same pattern as in a double-double. 1s can be in two places in the second column. They can be in two places in the third column. But between them they eliminate the possibility of any further ones appearing in the second and third columns. Therefore we can fill in pencil marks for the two remaining cells in which they can still be found within that 9-block:

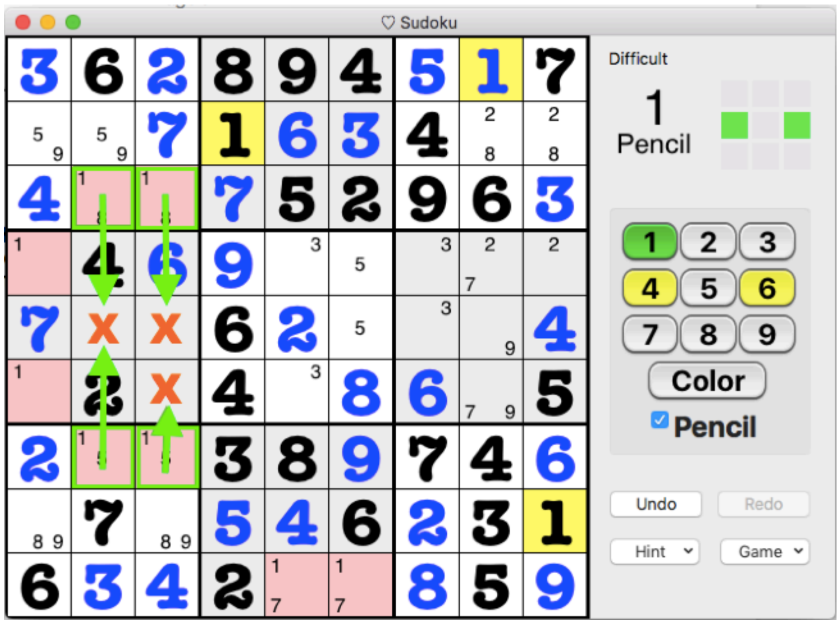


Fig. 6: X-wing eliminations

Here’s another example of an x-wing strike that arose in the early stages of play:

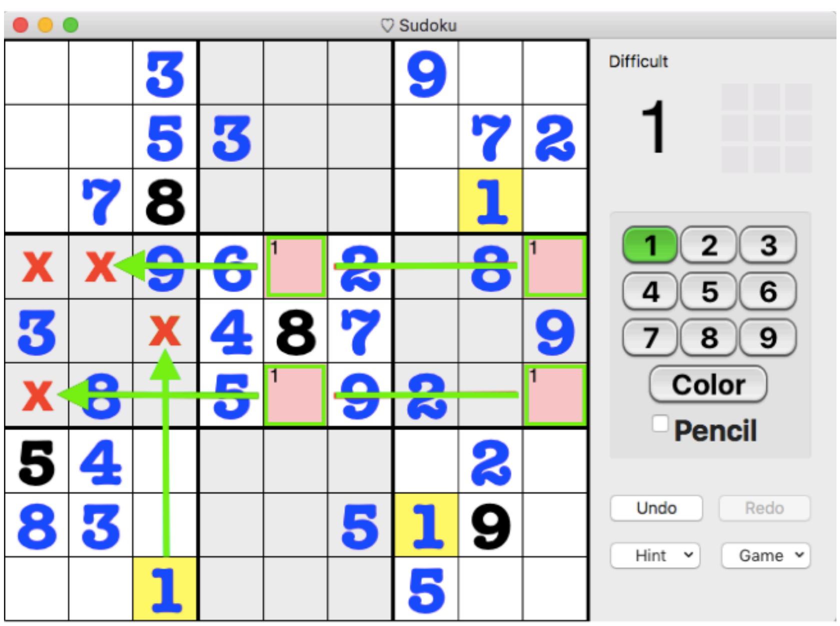


Fig. 7: Second x-wing example (work-along 2)

An x-wing occurs when four cells in a rectangular pattern contain the same digit — but only if no other cells in either row of the rectangle contain that digit OR no other cells in either column of the rectangle contain that digit. A row-based x-wing eliminates all other occurrences of the digit in its columns. A column-based x-wing eliminates all other occurrences of the digit in its rows.

Here, the x-wing is column-based instead of row-based like the one in Fig. 5, meaning the only 1s in columns 6 and 9 are confined to rows 4 and 6. Between the x-wing and the supplied 1 digit in the bottom-left 9-block we see that the 1 in the middle-left 9-block can only go in the centre cell of that block.

3. The skyscraper scrape

You may be amazed to know that a skyscraper is just a lop-sided x-wing in disguise. And, technically speaking, you may be even more surprised to learn that a lop-sided x-wing is just a form of finned sashimi. Sudoku vocabulary is an Alice-in-Wonderland sort of thing. We’ll stick with the term skyscraper and be glad we don’t have to wrap our minds around the fish-speak.

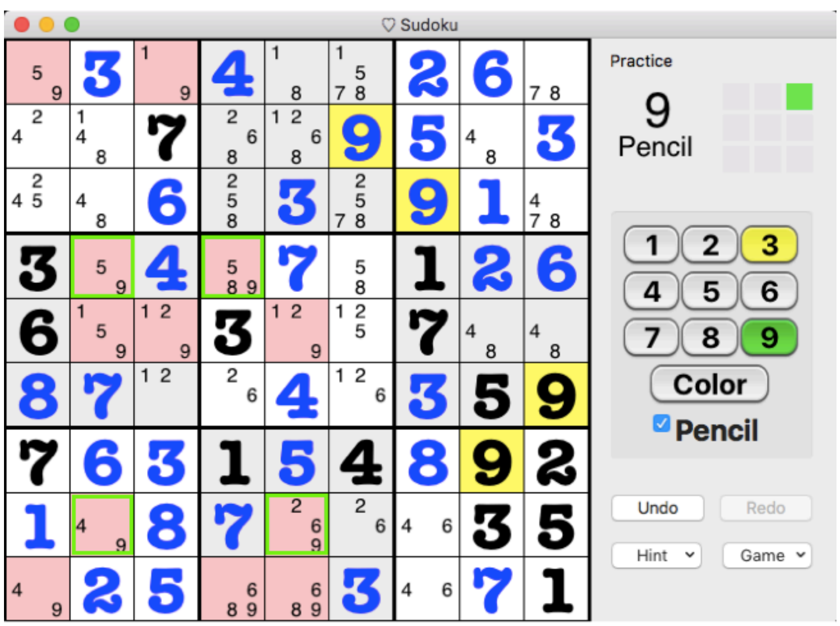


Fig. 8: Skyscraper on 9s (work-along 3)

Unlike an x-wing, a skyscraper can only eliminate candidates that are in the direct line of fire of both cells on the skyscraper’s lop-sided edge. This is illustrated in Fig. 9:

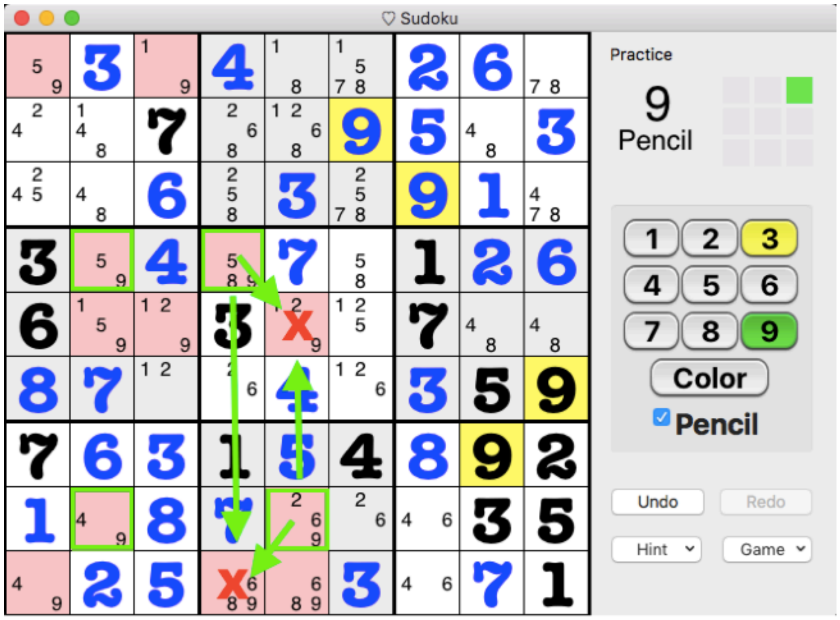


Fig. 9: Skyscraper eliminations

While technically the skyscraper pattern can occur in one, two, and three 9-blocks, in practice it’s only useful when each of the four cells is in a separate 9-block.

Here’s another example, this time with the skyscraper upright instead of on its side:

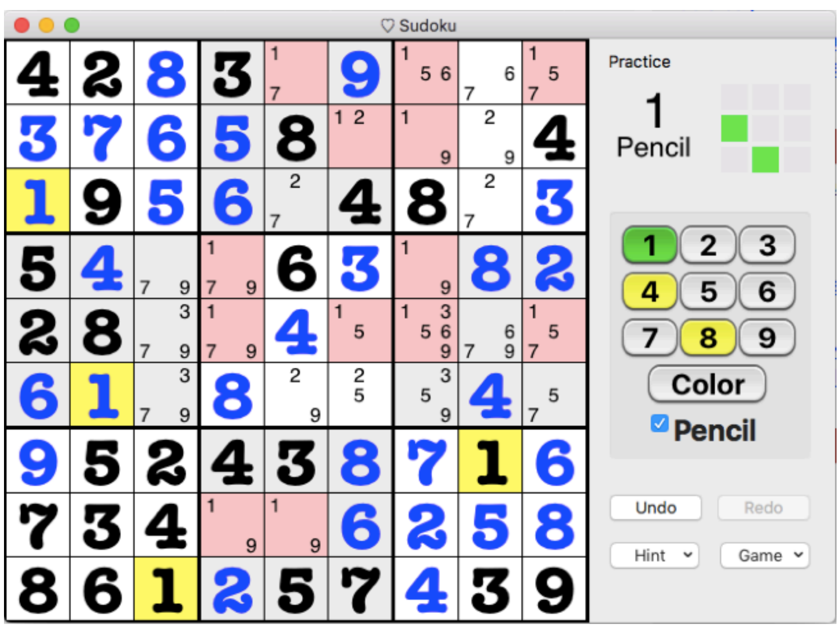


Fig. 9: Another skyscraper example (work-along 4)

I didn’t mark it this time; see if you can spot it. The eliminations it reveals lead quickly to a win.

A skyscraper occurs when four cells in a lop-sided rectangular pattern contain the same digit — but only if no other cells in either row of the rectangle contain that digit OR no other cells in either column of the rectangle contain that digit. A skyscraper eliminates all other occurrences of the digit within both 9-blocks that contain the non-right-angle corner cells.

✣ ✧ ✣ ✧ ✣ ✧ ✣

This completes our survey of techniques useful for working without pencil marks and with rule-limited pencil marks. Full-capability digital apps, such as Heart Sudoku on MacOS and IOS and Simple Sudoku on Windows make it practical to work with all possible pencil marks initially showing. The player then uses logic to eliminate candidates wherever possible, and uses highlighting to search for patterns among the remaining pencil marks. When solving in this manner a myriad of other sudoku techniques become practical, such as swordfish, jellyfish, unique rectangles and chains. But the vast majority of web page, newspaper and sudoku book puzzles can be solved with only the tactics we've covered in these two tutorials.

To conclude in keeping with the first tutorial, here's a printable practice sudoku that incorporates all three of the techniques covered in this tutorial, hidden double, x-wing, and skyscraper:

		5					7	4
	1		6				3	
	7				1			
	6			3	7	2		
		1	2	5			6	
			1				4	
	3				9		8	
4	2					7		

Appendix: Instructional puzzles

(Print, manually copy on paper or enter in to a sudoku program.)

3		2				5	1	
		7		6	3			
4			7					3
		6	9					
7				2				4
					8	6		
2					9			6
			5	4		2		
	3	4				8		9

work-along 1

		3				9		
		5	3				7	2
	7						1	
		9	6		2		8	
3			4		7			9
	8		5		9	2		
	4						2	
8	3				5	1		
		1				5		

work-along 2

	3		4			2	6	
					9	5		3
		6		3		9	1	
		4		7			2	6
8	7			4		3		
	6	3		5		8		
1		8	7					
	2	5			3		7	

work-along 3

		8			9			
3	7	6	5					
1		5	6					3
	4				3		8	2
				4				
6	1		8				4	
9					8	7		6
					6	2	5	8
			2			4		

work-along 4